MATRIX ALGEBRA--TOOL FOR SIMPLIFYING CALCULATIONS

WHAT IS A MATRIX

- DEFINITION: ANY ARRAY OF NUMBERS

Examples of Matrices

$$\begin{bmatrix} 10 & 2 \\ 1 & 10 \end{bmatrix} \qquad \begin{bmatrix} 1 & 100 \\ 2 & 50 \\ 3 & 25 \\ 4 & 12 \end{bmatrix}$$

$$[12 \quad 14 \quad 18 \quad 20]$$

$$[10]$$

Matrices

- MATRICES ARE DENOTED WITH UPPER CASE LETTERS
- ELEMENTS OF MATRIX ARE DENOTED WITH SUBSCRIPTED LOWER CASE LETTERS
- EXAMPLE:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

- a_{11} = ELEMENT IN FIRST ROW, FIRST COLUMN

ORDER OF A MATRIX

REFERS TO HOW MANY ROWS AND COLUMNS IT CONTAINS

- ORDER OF A MATRIX IS rxc
- a_{rc}

ORDER OF A MATRIX

- IT IS USEFUL TO WRITE THE ORDER OF A MATRIX AT THE LOWER LEFT CORNER TO KEEP TRACK OF THE ORDER

$$\begin{bmatrix} 1 & 2 & 3 \\ 10 & 12 & 14 \end{bmatrix}_{2x3}$$

- VECTOR = IS A MATRIX WITH ONE ROW OR COLUMN

$$[8 \quad 249 \quad 6 \quad 10]_{1x4}$$

Vector

- A VECTOR IS SYMBOLIZED BY A LOWER CASE LETTER WITH A TILDA
-
-
-
- a~ = [8 249 6 10]
- 1X4
-
-
- VECTORS ARE USEFUL FOR DESCRIBING A SYSTEM, OR ITEM WITH MORE THAN ONE CHARACTERISTIC

Vectors (examples)

- INDIVIDUAL [HT WT AGE]
-
- POPULATION [YOUNG AGE1 AGE2 AGE3]
-
- LANDSCAPE [%FARMLAND %TREES %WETLAND]

SCALAR

- A SCALAR = A MATRIX WITH JUST ONE NUMBER
-
-
- $[a_{ii}]_{1x1}$ A 1x1 MATRIX
-
- A SCALAR IS DENOTED BY LOWER CASE LETTER
-

MATRIX OPERATIONS

- ADDITION

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 100 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 3 & 0 \\ 2 & 1 & 10 \end{bmatrix}$$

$$\text{THEN } A + B = \begin{bmatrix} 1+5 & 0+3 & 2+0 \\ 3+2 & 100+1 & 4+10 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 2 \\ 5 & 101 & 14 \end{bmatrix}$$

- TWO MATRICES MUST HAVE SAME ORDER TO BE ADDED

Matrix Operations

- SUBTRACTION== SAME AS ADDITION
-
- MUST HAVE EQUAL ORDER
-
-
-
- SUBTRACT CORRESPONDING ELEMENTS
-

MATRIX MULTIPLICATION

- SCALAR MULTIPLICATION
- MULTIPLY EACH ELEMENT IN MATRIX BY A SCALAR
- $a \times A = [5] \begin{bmatrix} 5 & 3 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} 25 & 15 \\ 40 & 5 \end{bmatrix}$
- HIGHER ORDER MULTIPLICATION
- WARNING:--DO NOT SIMPLY MULTIPLY LIKE ELEMENTS
- MULTIPLY ELEMENTS OF EACH ROW WITH CORRESPONDING ELEMENTS IN EACH COLUMN AND ADD THEM UP

MATRIX MULTIPLICATION

- RULES
- COLUMNS OF FIRST MATRIX MUST EQUAL ROWS OF SECOND MATRIX
- [R C] 2X3 3X2 2X2

MATRIX MULTIPLICATION

- EXAMPLE:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 5 \end{bmatrix}_{2X3} \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}_{3X2}$$

- THEN:

$$A \text{ X } B = \begin{bmatrix} 1+4+3 \\ 3+8+15 \end{bmatrix} = \begin{bmatrix} 0+2+0 \\ 0+4+5 \end{bmatrix} \quad \begin{bmatrix} 8 & 2 \\ 26 & 9 \end{bmatrix}_{2X2}$$

MATRIX MULTIPLICATION

- NOTE: A X B DOES NOT EQUAL B X A

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1+0 & 2+0 & 1+0 \\ 2+3 & 4+4 & 2+5 \\ 3+0 & 6+0 & 3+0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 5 & 8 & 7 \\ 3 & 6 & 3 \end{bmatrix}$$

3X2 X 2X3 = 3X3

MATRIX MULTIPLICATION

- RULES: A: NUMBER OF COLUMNS OF FIRST MATRIX MUST EQUAL NUMBER OF ROWS OF SECOND MATRIX
-
-
- A B WILL PRODUCE lXn MATRIX
- lxm mxn
-
- NOTE : ROWS OF FIRST MATRIX AN COLUMNS OF SECOND MATRIX NEED NOT MATCH

MULTIPLICATION OF MATRIX BY A VECTOR

- EXAMPLE:

$$\tilde{a} = \begin{bmatrix} 2 & 5 \end{bmatrix}_{1\times 2} \qquad B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}_{2\times 2} \qquad \tilde{b} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}_{2\times 1}$$

$$\tilde{a} \times B = \begin{bmatrix} 4+20 & 2+15 \end{bmatrix}_{1\times 2}$$

$$= \begin{bmatrix} 24 & 17 \end{bmatrix}_{1\times 2}$$

MULTIPLICATION OF VECTOR BY A MATRIX

$$B \times b\sim = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad \begin{bmatrix} 4+5 \\ 8+15 \end{bmatrix} \quad \begin{bmatrix} 9 \\ 23 \end{bmatrix}$$

2x2 2x1 2x1 2x1

PROPERTIES OF MATRIX ALGEBRA

- (A+B) = (B+A) ADDITION IS COMMUTATIVE (INDEPENDENCE OF ORDER IN WHICH THE ELEMENTS ARE TAKEN)
- (A-B) DOES NOT EQUAL (B-A)
- (AxB) DOES NOT EQUAL (BxA) MULTIPLICATION IS NOT COMMUTATIVE
- axB = Bxa MULTIPLICATION WITH A SCALAR IS COMMUTATIVE

LESLIE MATRIX

- SO WHAT DOES ALL THIS HAVE TO DO WITH POPULATION MODELS??
-
- MATRIX ALGEBRA IS A CONVENIENT WAY TO KEEP TRACK OF AGE STRUCTURE DURING POPULATION GROWTH
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LESLIE MATRIX

- LET'S TAKE A LOOK AT HOW WE WOULD DO THIS
- EXAMPLE 1:
- SUPPOSE THAT MAMMAL POPULATION WITH 4 AGE CLASSES

AGE	N_X	M_X	S_X
0	1750	0	0.1
1	100	5	0.6
2	100	15	0.3
3	50	10	0.0

- Where: N_x = number; M_x = fecundity; S_x = Survival

LESLIE MATRIX

- LET N_{0t} = NUMBER OF INDIVIDUALS AGED 0 AT TIME t
-
- LET N_{1t} = NUMBER OF INDIVIDUALS AGED 1 AT TIME t
-
- LET N_{2t} = NUMBER OF INDIVIDUALS AGED 2 AT TIME t
-
- LET N_{3t} = NUMBER OF INDIVIDUALS AGED 3 AT TIME t
-

LESLIE MATRIX

- $N_{0t+1} = M_0N_{0t} + M_1N_{1t} + M_2N_{2t} + M_3N_{3t}$
-
- $N_{1t+1} = N_{0t}S_0 + 0 + 0 + 0$
-
- $N_{2t+1} = 0 + N_{1t}S_1 + 0 + 0$
-
- $N_{3t+1} = 0 + 0 + N_{2t}S_2 + 0$
-
-
- NOTE: ASSUMES THAT POPULATION REPRODUCES, THEN DIES.
-

LESLIE MATRIX

- PUT THESE COEFFICIENTS IN MATRIX FORM

$$\begin{bmatrix} 0 & 5 & 15 & 10 \\ 0.1 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix} \times \begin{bmatrix} 1750 \\ 100 \\ 100 \\ 50 \end{bmatrix} =$$

LESLIE MATRIX

$$\begin{bmatrix} 0(1750) + 5(100) + 15(100) + 10(50) \\ \\ 0.1(1750) + 0(100) + 0(100) + 0(50) \\ \\ 0(1750) + 0.6(100) + 0(100) + 0(50) \\ \\ 0(1750) + 0(100) + 0.3(100) + 0(50) \end{bmatrix} = \begin{bmatrix} 2500 \\ \\ 175 \\ \\ 60 \\ \\ 30 \end{bmatrix}$$