Chapter 5 – Mills (2007)

Exponential or Geometric Population Change

POPULATION GROWTH--EXPONENTIAL MODEL

-
- MODEL DESCRIBING THE "RATE OF INCREASE" OF ANIMAL POPULATIONS FACING UNLIMITED RESOURCES
-
- REASONS FOR STUDYING POPULATION GROWTH
-
- 1. COMPARE RATES OF GROWTH AMONG POPULATIONS TO:
-
- INDEX DEMOGRAPHIC VIGOR-demography?
- INDEX HABITAT POTENTIAL TO SUPPORT A SPECIES
-
- 2. THEORETICAL MODELS OF POPULATION GROWTH ALLOW US TO PREDICT ANIMAL RESPONSES TO CONTROL EFFORTS OR HARVEST

EXPONENTIAL

- SEVERAL THEORETICAL MODELS FOR THE FORM OF POPULATION INCREASE
-
- EXPONENTIAL GROWTH--APPLIES TO ANIMAL POPULATIONS FACING UNLIMITED RESOURCES
-
- UNDER SUCH CONDITIONS POPULATIONS INCREASE AT AN EVER INCREASING RATE
-
- So, why use this model??

DISCRETE MODEL

- dN/dt = B(TOTAL NUMBER OF BIRTHS) - D(TOTAL NUMBER OF DEATHS)
-
- R_B = "PER HEAD" BIRTH RATE
-
-
- = # BORN SURVIVING TO AGE 1
- # ADULTS
-
- R_D = "PER HEAD" DEATH RATE
-
-
- = # ADULTS (AGE 1+) DYING
- # ADULTS
-
-
- $dN/dt = R_B N - R_D N$

DISCRETE MODEL

- SUPPOSE THAT dt = 1 UNIT
-
- $dN = R_B N - R_D N$
-
- $dN = N_1 - N_0$
-
- $N_1 - N_0 = R_B N_0 - R_D N_0$
-
- $N_1 = N_0 + R_B N_0 - R_D N_0$
-
- $N_1 = N_0 (1 + R_B - R_D)$

DISCRETE MODEL

- FINITE RATE OF INCREASE = R = LAMBDA
-
- $N_1 = N_0R$
-
- $N_2 = N_1R = (N_0R)R = N_0R^2$
-
- $N_t = N_0R^t$

DERIVING R

-
- A. FROM BIRTH AND DEATH SCHEDULES
-
- B. FROM CENSUS DATA
-

$$R = \frac{N_{t+1}}{N_t}$$

-

$$R = 1 + R_B - R_D$$

MEANING OF R

-
- "PER CAPITA" RATE OF INCREASE
-
- IF R = 1.8
-
- THEN FOR EVERY INDIVIDUAL IN POPULATION AT t_i THERE WILL BE 1.8
-
- AT TIME t_{i+1}
-

CONTINUOUS MODEL

- IMAGINE SHRINKING OUR TIME INTERVALS SO SMALL THAT BIRTHS AND DEATHS ARE NOW OCCURRING AT AN INSTANTANEOUS SCALE, YIELDING A CONTINUOUS PATTERN OF GROWTH
-
- WE CAN EXPRESS RATE OF CHANGE IN THE POPULATION AS A DIFFERENTIAL EQUATION

CONTINUOUS MODEL

- $dN/dt = N(B) - N(D)$
-
- WHERE N = NUMBER OF ANIMALS IN POPULATION
-
- B = INSTANTANEOUS "PER HEAD" BIRTH RATE
-
- D = INSTANTANEOUS "PER HEAD" DEATH RATE
-

CONTINUOUS MODEL

- dN/dt = N(B - D)
-
- dN/dt = N(r)
-
- dN/dt = rN; hence, r = dN/dt/N
-
- WHERE r IS THE "PER HEAD" INSTANTANEOUS RATE OF INCREASE
-
- NOTE: POPULATION RATE OF INCREASE (dN/dt) VS. "PER HEAD RATE OF INCREASE

CONTINUOUS MODEL

- INTEGRATION:
-
- $N_t = N_0e^{rT}$
-
- WHERE: e = 2.7183 (APPROX) = EULER'S CONSTANT AND HAS THE PROPERTY THAT ITS NATURAL LOGARITHM IS EQUAL TO 1.0.
-

FINITE VS. INSTANTANEOUS

- $N = N_0R^t$ $N = N_0e^{rt}$
-
- $R = e^r$
-
- $Ln(R) = r$

SAMPLE CALCULATIONS

-
- 1. ASSUMING AN INSTANTANEOUS RATE OF INCREASE OF 0.69 WHAT IS POPULATION AT DIFFERENT TIME INTERVALS GIVEN 100 INTRODUCED ANIMALS?
-
- $N_t = 100e^{0.69t}$
-
- t N_t
-
- 0 100
-
- 1 200
-
- 2 400
-
- 3 800

SAMPLE CALCULATIONS

- ASSUMING 20 SHARP-TAILED GROUSE INTRODUCED IN 1990, 80 IN 1993,
-
- WHAT IS r?
-
- $80 = 20e^{r3}$
-
- $Ln(80) = Ln(20) + 3r$
-
- $4.38 = 2.99 + 3r$
-
- $3r = 4.38 - 2.99$
-
- $3r = 1.39$
-
- $r = 1.39/3 = 0.46$
-

FINITE VS. INSTANTANEOUS

- DEER POPULATION IS INCREASING AT A FINITE RATE OF INCREASE OF 1.6, WHAT IS THE INSTANTANEOUS RATE OF INCREASE?
-
- R = 1.6
-
- Ln(1.6) = 0.47
-
- 0.47 = r
-
- RATE OF INCREASE MAY BE EXPRESSED AS FINE RATE = e^r OR R
-
- OR INSTANTANEOUS RATE = r
-

ADVANTAGES OF INSTANTANEOUS RATE

- 1. CENTERED ON ZERO (WHEREAS e^r CENTERED ON 1)
- RATES OF INCREASE AND DECREASE ARE DIRECTLY COMPARABLE FROM CHANGES IN SIGN
- $r = 0.5$ $e^r = 1.649$
- $r = -0.5$ $e^r = 0.607$
- 2. r EASILY CONVERTS FROM ONE UNIT OF TIME TO ANOTHER
- IF r IS MEASURED FOR AN ANNUAL PERIOD, THEN THE DAILY RATE OF INCREASE = r/365
- WEEKLY RATE OF INCREASE = r/52

COMPUTING R VS r

- A. $R = N_{t+1}/N_t$ $R = e^r$ $r = Ln(R)$
- B. $N = N_0e^{rt}$
- $Ln(N) = Ln(N_0) + rt$
- WHICH IS EQUIVALENT TO FORMULA FOR A STRAIGHT LINE:
- $(Y = a + bX)$

MEASURES OF RATE OF INCREASE

- r_m = r_{max} = INTRINSIC RATE OF INCREASE WHEN NO RESOURCES ARE LIMITING
-
- DETERIMINED BY THE SPECIES GENETIC POTENTIAL
-

MEASURES OF RATE OF INCREASE

- r = OBSERVED RATE OF INCREASE IS THE EXPONENTIAL RATE OVER WHICH A POPULATION INCREASES OVER ANY OBSERVED PERIOD OF TIME
-
- r INCORPORATES ALL LIMITING FACTORS THAT MAY BE EXPERIENCED DURING THAT TIME
-
- DOES NOT IMPLY CONSTANCY

MEASURES OF RATE OF INCREASE

- r_S = SURVIVAL-FECUNDITY RATE OF INCREASE
-
- r_s IS THE EXPONENTIAL RATE AT WHICH A POPULATION WOULD INCREASE IF IT HAD A STABLE AGE DISTRIBUTION APPROPRIATE TO ITS CURRENT SCHEDULES OF AGE-SPECIFIC SURVIVAL AND FECUNDITY—FROM LIFE TABLE.
-
- r_s USUALLY DIFFERS FROM THE ACTUAL RATE OF INCREASE BECAUSE AN AGE DISTRIBUTION IS SELDOM STABLE. HOWEVER, THE r_s RATE OF INCREASE REVEALS A POPULATION'S CURRENT CAPACITY TO INCREASE (BECAUSE IT IS BASED ON THE AGE STRUCTURE OF THE POPULATION.
-

MEASURES OF RATE OF INCREASE

- r_m, r, AND r_s ALL MAY BE USED AS INDICES OF DEMOGRAPHIC VIGOR TO
-
- COMPARE A POPULATION'S WELL BEING BASED ON, FOR EXAMPLE:
-
  - FAT RESERVES, BODY SIZE, SEX RATIO
-
- INDICES ARE USEFUL TO THE EXTENT THAT THEY INDEX r

MEASURES OF RATE OF INCREASE

- r_p POTENTIAL RATE OF INCREASE
-
- DEMOGRAPHIC VIGOR CAN BE MODIFIED FOR HARVESTED POPULATIONS.
-
- IF HARVESTING IS ACCOMPLISHED "QUANTITATIVELY", SO THAT r_s IS STABLE AT 0, THEN THE VALUE OF INTEREST BECOMES RATE OF INCREASE IF HARVESTING IS TERMINATED.
-
- r_p OF HARVESTED POPULATION CAN BE MEASURED DIRECTLY BY HALTING HARVESTING OR CAN BE ESTIMATED FROM A LIFE TABLE PARTITIONED INTO RATES OF HARVESTING MORTALITY AND NATURAL MORTALITY SO THAT YOU CAN REMOVE HARVEST MORTALITY AND ESTIMATE POTENTIAL RATE OF INCREASE.
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ASSUMPTIONS FOR EXPONENTIAL GROWTH MODEL

- 1. NO AGE STRUCTURE (ALL INDIVIDUALS ALIKE).
-
- 2. r IS CONSTANT (RESOURCES UNLIMITED DURING PERIOD MEASURED).

EXAMPLES OF EXPONENTIAL GROWTH

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- A. PHEASANTS ON PROTECTION ISLAND, WASHINGTON
-
- 400 ACRES OF IDEAL PHEASANT HABITAT
-
- NO PREDATORS
-
- MOSAIC OF WHEAT, BARLEY, OATS, GRASSLAND, PASTURE
-
- 8 PHEASANTS RELEASED IN 1938
-
- 1,500 PHEASANTS BY 1947

EXAMPLES OF EXPONENTIAL GROWTH

- INSTANTANEOUS RATE OF INCREASE --pheasants
-
- $1,500 = 8e^{r4}$
-
- $Ln(1,500) = Ln(8) + 4r$
-
- $7.31 = 2.07 + 4r$
-
- $4r = 7.31 - 2.07$
-
- $4r = 5.24$
-
- $r = 1.31$
-
- $R = 3.7$
-

EXAMPLES OF EXPONENTIAL GROWTH

- INTRODUCED REINDEER ON ST. MATHHEWS ISLAND
-
- 29 REINDEER INTRODUCED BY US COAST GUARD
-
- 6,000 REINDEER BY 1963

EXAMPLES OF EXPONENTIAL GROWTH

- $6{,}000 = 29e^{r19}$
-
- $\ln(6{,}000) = \ln(29) + 19r$
-
- $8.69 = 3.36 + 19r$
-
- $19r = 8.69 - 3.36$
-
- $19r = 5.33$
-
- $r = 0.28$
-
- $R = 1.32$

EXAMPLES OF EXPONENTIAL GROWTH

- DEER OF GEORGE RESERVE
-
- 6 DEER (4 DOES, 2 BUCKS) INTRODUCED IN 1928)--6 YEARS LATER 180 DEER
-
- 1967-- DRASTIC REDUCTION TO 10 ANIMALS AND 6 YEARS LATER AFTER NO HUNTING-212 DEER
-
- $r = Ln(N^6)- Ln(N_0)/t = Ln(180) - Ln(6)/6 = 5.19 - 1.79/6$
-
- $r = 0.57$
-
- $R =1.76$
-

Variation in Population Growth

- Demographic stochasticity—deviation in birth and death rates—more important at low population size
- Environmental stochasticity—arises from extrinsic factors (weather)
- Catastrophes and bonanzas—rare—outside the range of normal environmental stochasticity

Implications for Variation in Population Growth

- Increasing variation in growth rate causes geometric mean of population growth to be less than arithmetic mean
- Variation in growth rate can lead to population decline despite r values greater than zero
- $Rg = (R_1 * R_2 * R_3)^{1/2}$

Density-Independent Diffusion Approximation Method

- Used when missing estimates of growth rate
- Regress xi (transformed time interval) with yi (population change corrected for time interval)
- Slope = mean of r or rbar and standard error from regression analysis provides estimate of variance to generate CI for rbar